

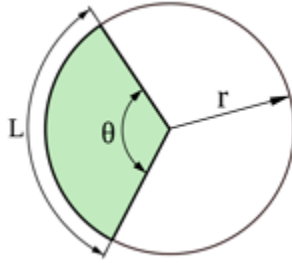
Geometry Formula Reference

Circle

Equation of Circle: $(x-h)^2 + (y-k)^2 = R^2$

Area $A = \pi R^2$

Circumference $C = 2\pi R = \pi * \text{Diameter} = \pi D$

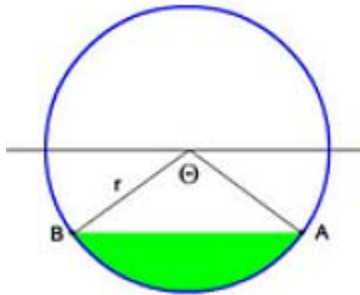


Length of Arc Defined by Angle θ (radians) $L = R\theta$

If Angle in Degrees $L = R\theta \frac{\pi}{180}$

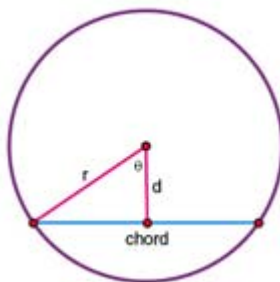
Area of Sector of Circle (green) Defined by Angle θ (radians) $A = \frac{1}{2} R^2 \theta$

If Angle in Degrees $A = \frac{1}{2} R^2 \theta \frac{\pi}{180}$



Area of circle Segment (green) Defined by Angle θ (radians) $A = \frac{1}{2} r^2 (\theta - \sin(\theta))$

If Angle in Degrees $A = \frac{1}{2} r^2 (\theta \frac{\pi}{180} - \sin(\theta))$



Length of the Chord $L = 2 * \sqrt{r^2 - d^2}$

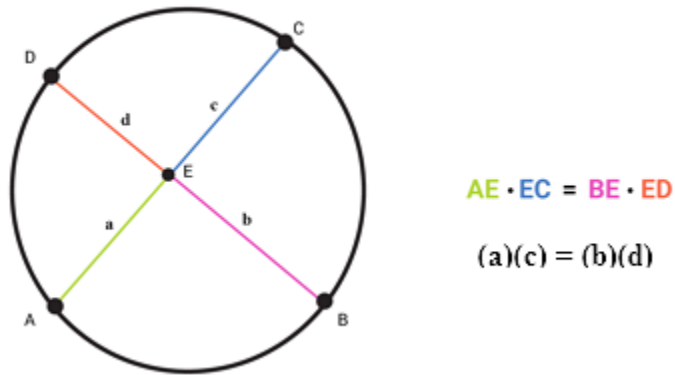
Or

$L = 2r * \sin(\theta)$

Circle

Theorems for Lines Intersecting a Circle

Theorem of Intersecting Chords

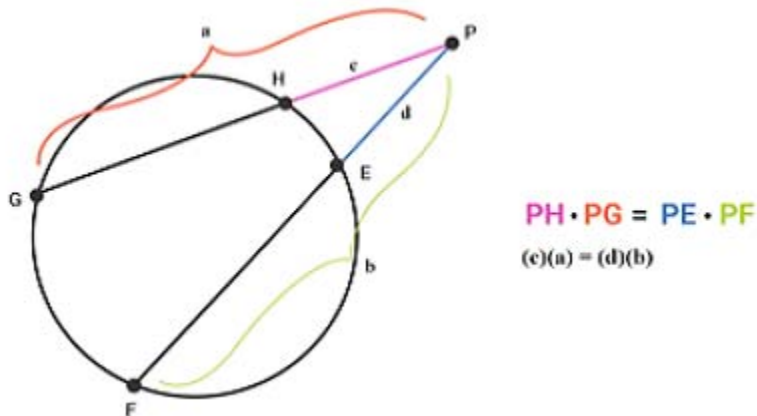


The intersection of 2 chord lines inside a circle result in 4 line lengths for the intersections at point E. The sub-divisions of each chord are always in the same product ratio. The line AE with length a times line EC with length c = line BE with length b times line ED with length d.

The 4 lengths are therefore related by the equation:

$$ac = bd$$

Theorem of Intersecting Secants



This theorem relates the 2 secant lines intersecting the circle. Note that secants always start outside the circle and intersect 2 points each. This time the relationship is to the outside segment and whole segment of each secant. The outside segment PH of length c times the entire length of PG = the outside segment PE of length d times the entire length of PF.

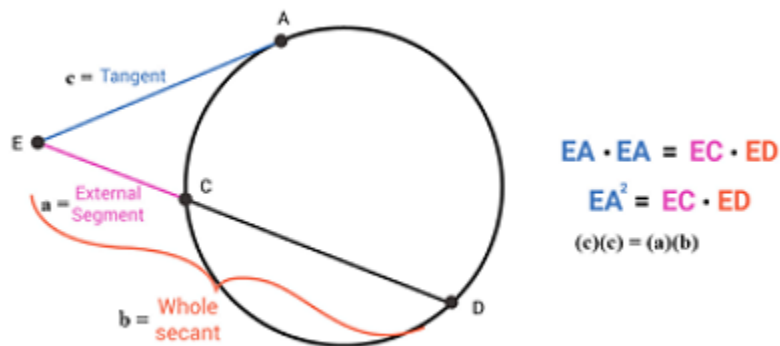
The 4 lengths are therefore related by the equation:

$$ca = db$$

Circle

Theorems for Lines Intersecting a Circle

Theorem of Intersecting Secant & Tangent



This time, the 2nd line is a tangent which only intersects with the circle at one point, so the secant product ratio is satisfied by the square of the tangent.

The 3 lengths are therefore related by the equation:

$$c^2 = ab$$

Parabola

Equation of Parabola: $y = a(x-h)^2 + k$ or $x = a(y-k)^2 + h$

Vertex at (h,k)

Regular Form: $y = ax^2 + bx + c$

Then $h = \frac{-b}{2a}$ $k = 4ac - b^2/4a$

Focus is at $(h, k+p)$ or $(h+p, k)$ where $p = \frac{1}{4a}$

Directrix is at $y = k - p$ or $x = h - p$

Ellipse

Standard Equation

With the center of the ellipse at (h, k)
and the major axis parallel to the x axis:

$a > b$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Length of major axis = $2a$

Major Axis Vertices are at $(h \pm a, k)$

Length of minor axis = $2b$

Minor Axis Vertices are at $(h, k \pm b)$

The Foci are at $(h \pm c, k)$, where $c^2 = a^2 - b^2$

Distance between Foci = $2c$

Also $c = \epsilon a$ (eccentricity * a), see below

With the center of the ellipse at (h, k)
and the major axis parallel to the y axis:

$a < b$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Length of major axis = $2a$

Major Axis Vertices are at $(h, k \pm a)$

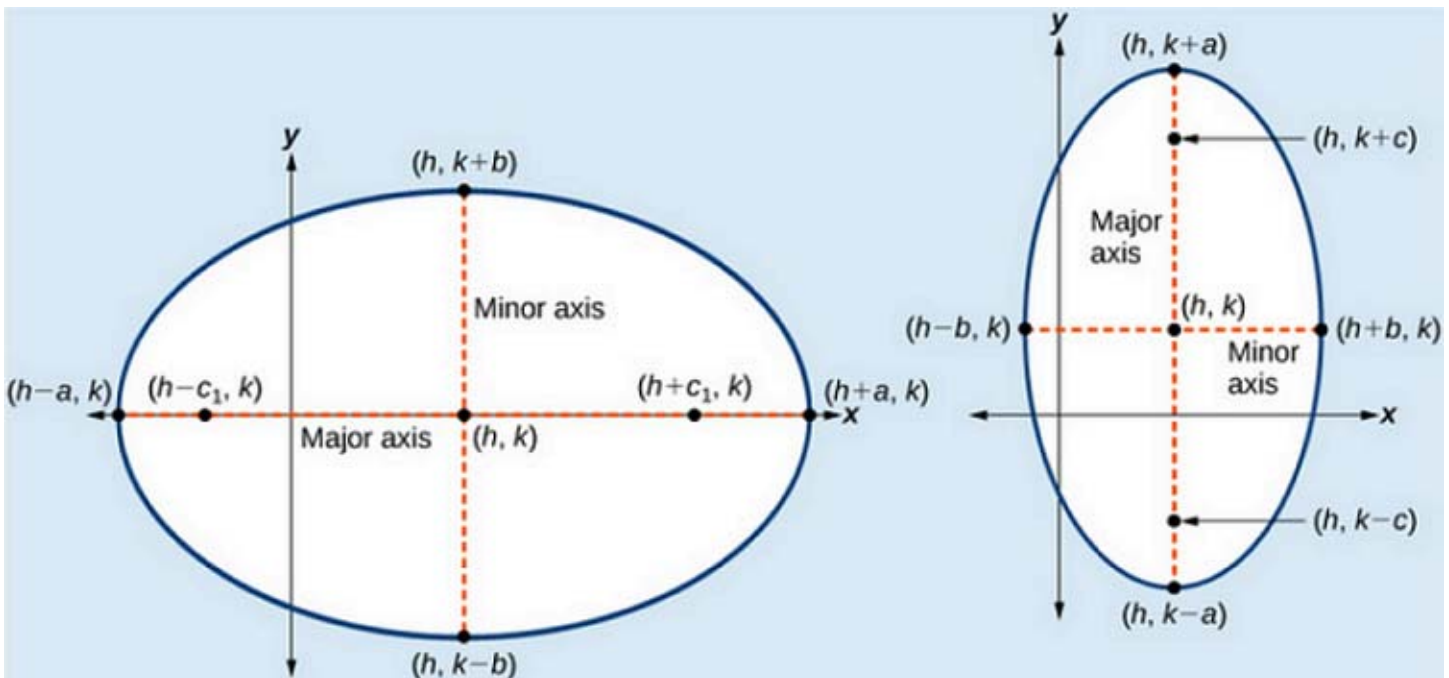
Length of minor axis = $2b$

Minor Axis Vertices are at $(h, k \pm b)$

The Foci are at $(h, k \pm c)$, where $c^2 = a^2 - b^2$

Distance between Foci = $2c$

Also $c = \epsilon a$ (eccentricity * a), see below



X & Y Major Axis Ellipses

The area of the ellipse: $A = \pi ab$

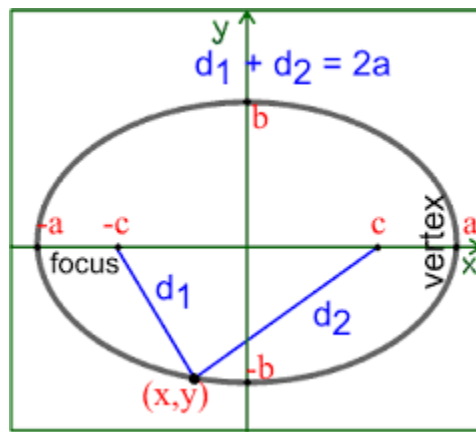
The perimeter of the ellipse: $P = 2\pi \frac{\sqrt{a^2+b^2}}{2}$

The eccentricity of ellipse: $\epsilon = \frac{c}{a}$ ($\epsilon a = c$)

The directrices are at $x = \{h \pm a^2/c, k\}$, for $a > b$
 $y = \{h, k \pm b^2/c\}$, for $a < b$

The length of Latus Rectum: $L_{lr} = 2b^2/a$

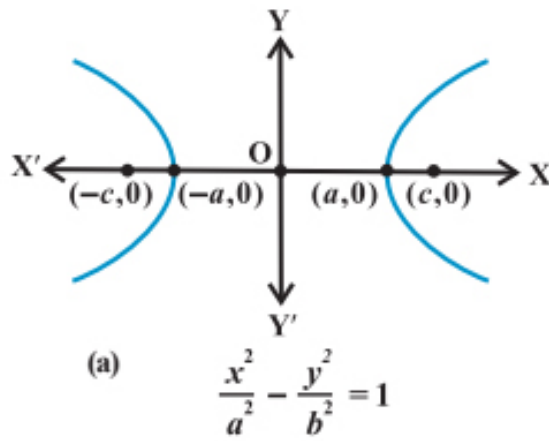
Ellipse



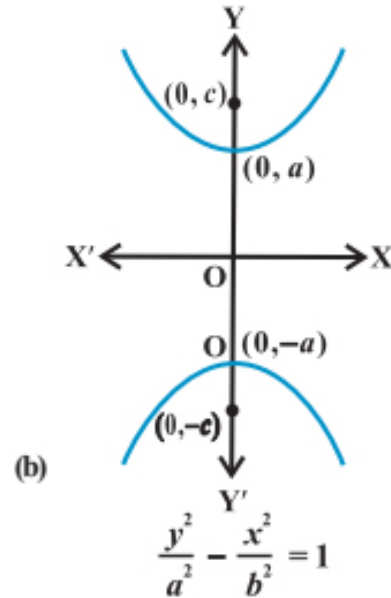
For any ellipse, 2 lines from each focus to a single point on the ellipse will have lengths d_1 and d_2 where the sum will always have $d_1 + d_2 = 2a$ for all points on the ellipse.

Hyperbola

Fig. 3



For $a > b$



For $a < b$

For any center $\neq$ origin, the standard equations become:

For $a > b$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse Axis: $y = k$

Conjugate Axis: $x = h$

Center at (h, k)

Focus to Origin Distance: $c^2 = a^2 + b^2$
Also $c = \epsilon a$ (eccentricity $\cdot a$), see below

Foci at $(h \pm c, k)$

Vertices at $(h \pm a, k)$

Slopes of Asymptotes $= \pm \frac{b}{a}$

Asymptote Equations:

$$y = +\frac{b}{a}(x-h)+k \text{ \& } y = -\frac{b}{a}(x-h)+k$$

Eccentricity: $\epsilon = \frac{c}{a}$

Direction of Opening = **Left & Right**

The length of Latus Rectum: $L_{lr} = 2b^2/a$

Directrices: $y = \pm a^2/c = \pm a^2/\sqrt{a^2 + b^2}$

For $a < b$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Transverse Axis: $x = h$

Conjugate Axis: $y = k$

Center at (h, k)

Focus to Origin Distance: $c^2 = a^2 + b^2$
Also $c = \epsilon a$ (eccentricity $\cdot a$), see below

Foci at $(h, k \pm c)$

Vertices at $(h, k \pm a)$

Slopes of Asymptotes $= \pm \frac{a}{b}$

Asymptote Equations:

$$y = +\frac{a}{b}(x-h)+k \text{ \& } y = -\frac{a}{b}(x-h)+k$$

Eccentricity: $\epsilon = \frac{c}{a}$

Direction of Opening = **Up & Down**

The length of Latus Rectum: $L_{lr} = 2b^2/a$

Directrices: $x = \pm a^2/c = \pm a^2/\sqrt{a^2 + b^2}$

Hyperbola

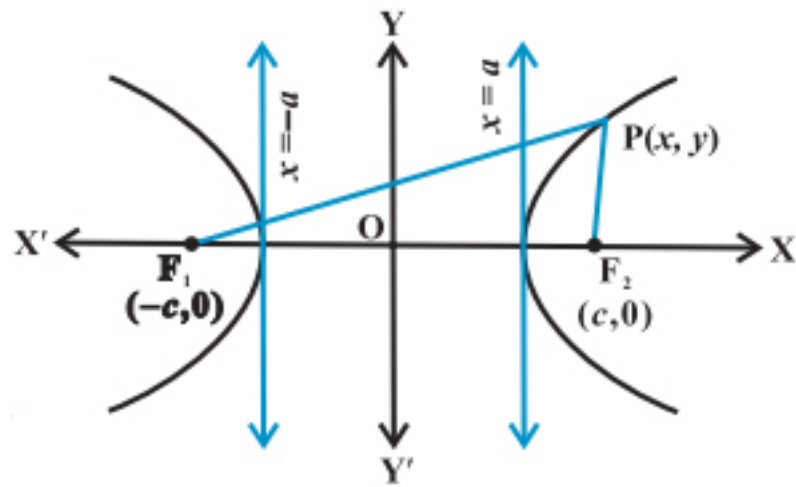


Fig. 4

For any point, $P(x,y)$, on either part of a hyperbola, the distance to the opposite focus (PF_1) minus the distance (PF_2) to the closest focus shall always $= 2a$.