

TRIGONOMETRY PART 2

(APPLICATIONS OF TRIGONOMETRY)

➤ Right Triangles

1. Angle of Elevation
2. Angle of Depression

➤ Oblique Triangles

1. Law of Sine
2. Law of Cosine



OBJECTIVES

At the end of this topic, the students would be able to.

- Apply trigonometric functions on worded problems.
- Solve problems relating to angle of elevation and depression.
- Execute mastery on the law of sine and cosine.

RELATIONSHIP BETWEEN TWO GIVEN ANGLES

Given two angles namely $\angle A$ and $\angle B$:

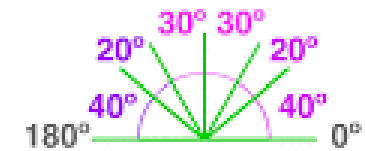
Complementary Angles $\angle A + \angle B = 90^\circ$

Supplementary Angles $\angle A + \angle B = 180^\circ$

Explementary Angles $\angle A + \angle B = 360^\circ$

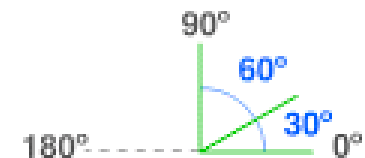
adjacent angles

Two angles immediately next to each other.



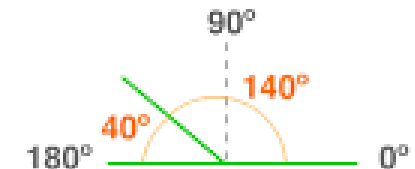
complementary angles

Two angles whose sum is 90° .



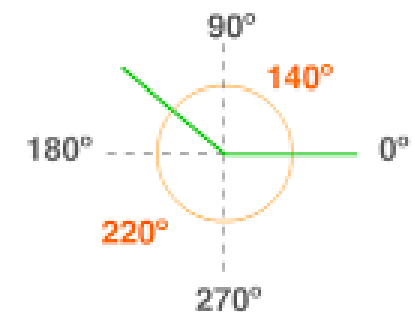
supplementary angles

Two angles whose sum is 180° .



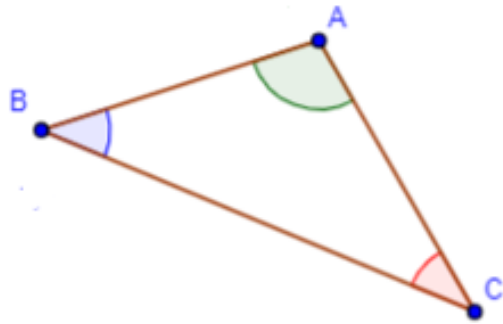
* conjugate or explementary angles

Two angles whose sum is 360° .



INTERIOR AND EXTERIOR ANGLES OF TRIANGLES

Sum of Angles in a Triangle

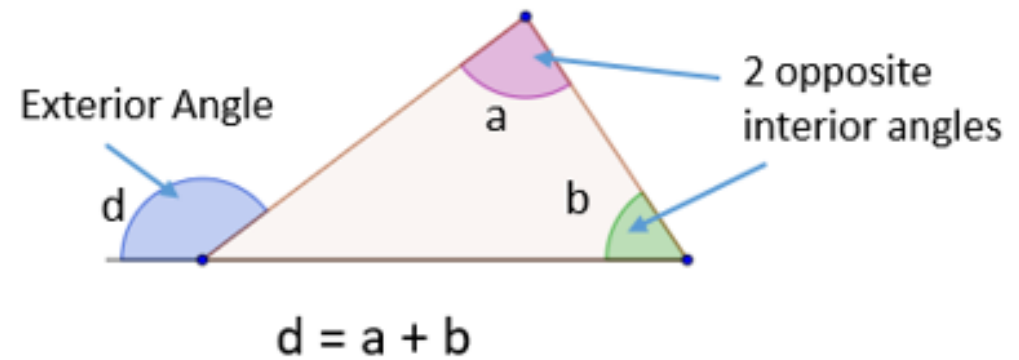


$$\angle A + \angle B + \angle C = 180^\circ$$

The sum of the angles in a triangle is always 180°

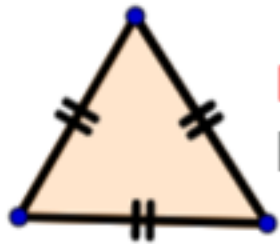
Exterior Angle Theorem

The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

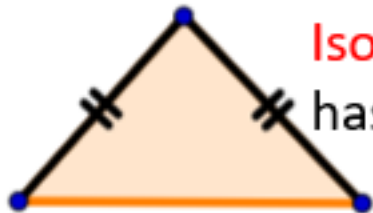


Types of Triangles

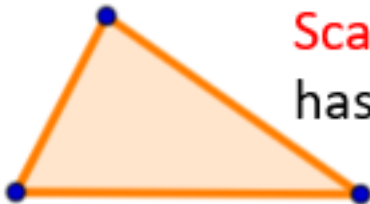
By Side



Equilateral Triangle
has three equal sides

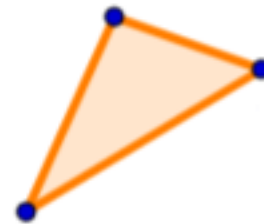


Isosceles Triangle
has two equal sides

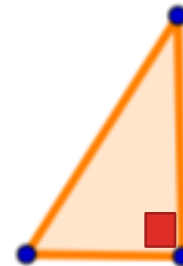


Scalene Triangle
has no equal sides

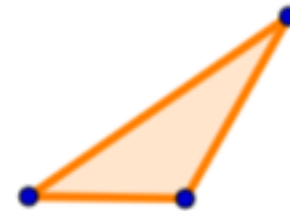
By Angle



Acute triangle
has three angles $< 90^\circ$



Right triangle
has one angle $= 90^\circ$



Obtuse triangle
has one angle $> 90^\circ$

APPLICATION ON RIGHT TRIANGLES

- A. Pythagorean Theorem
- B. Six Trigonometric Functions of the acute angle
- C. Problems involving angle of elevation and depression

The Six Trigonometric Functions

1. $\sin \theta = \frac{a}{c}$

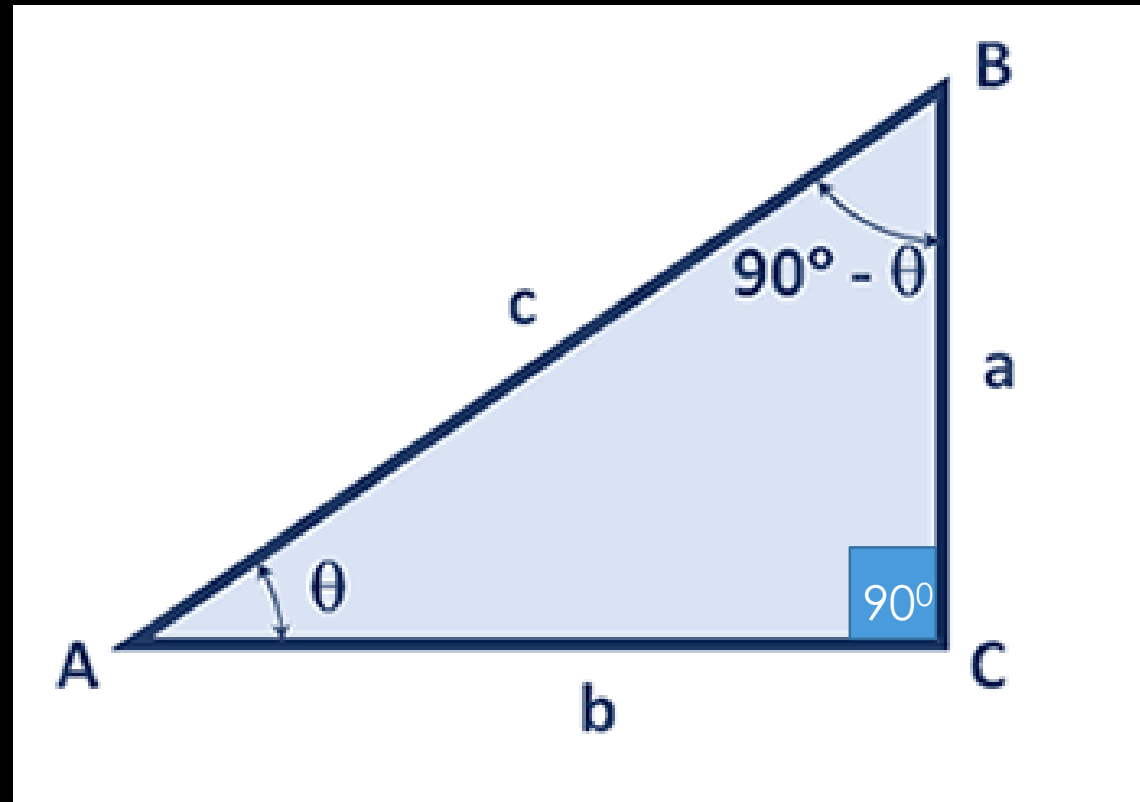
2. $\cos \theta = \frac{b}{c}$

3. $\tan \theta = \frac{a}{b}$

4. $\csc \theta = \frac{c}{a}$

5. $\sec \theta = \frac{c}{b}$

6. $\cot \theta = \frac{b}{a}$



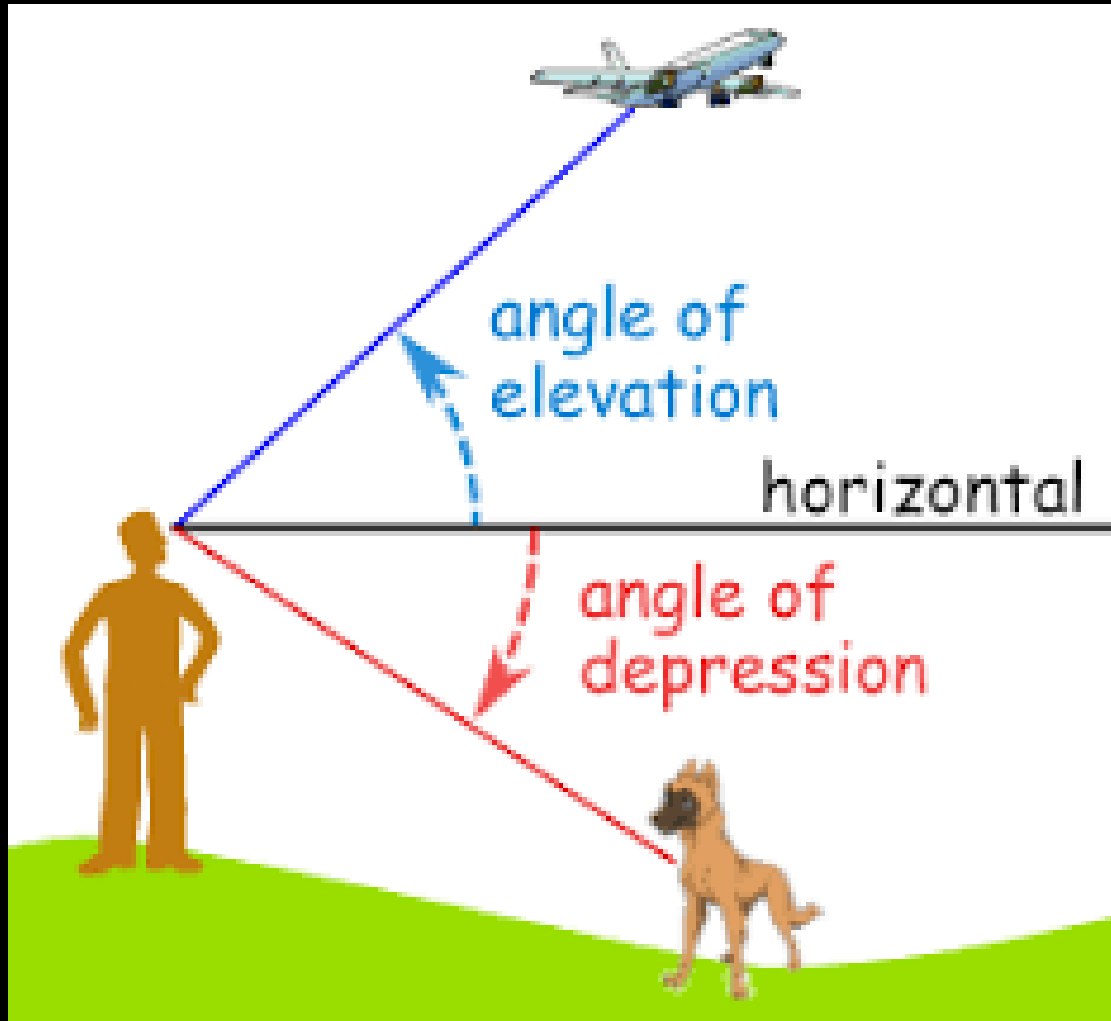
PROBLEMS INVOLVING ANGLE OF ELEVATION ANGLE OF DEPRESSION

How to solve word problems that involve angle of elevation or depression?

Step 1: Draw a sketch of the situation.

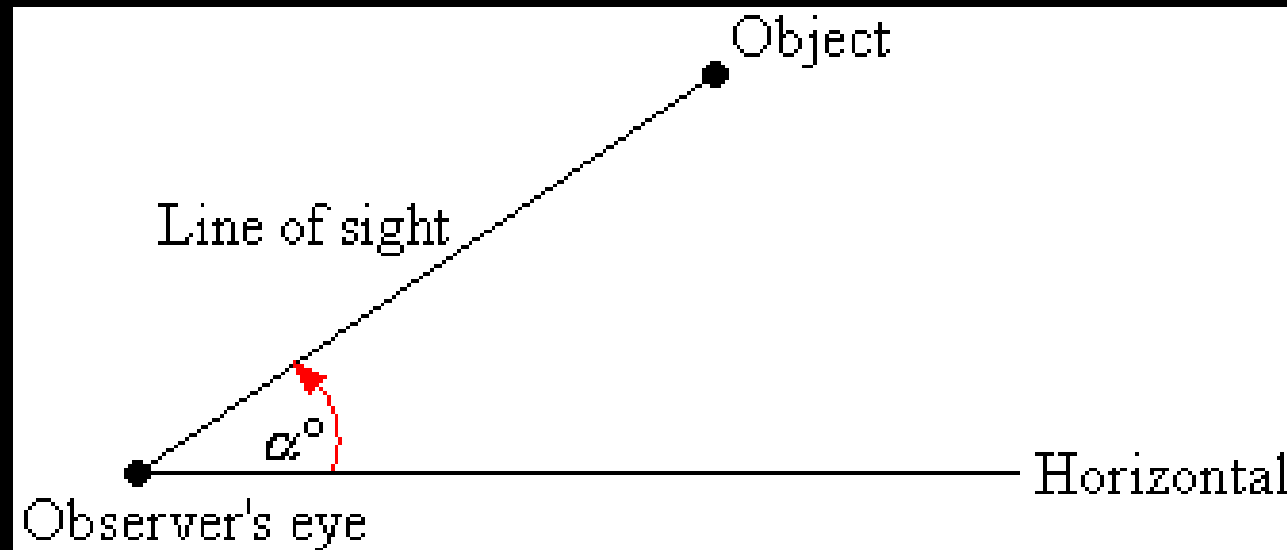
Step 2: Mark in the given angle of elevation or depression.

Step 3: Use trigonometry to find the required missing length



ANGLE OF ELEVATION

The **angle of elevation** of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).

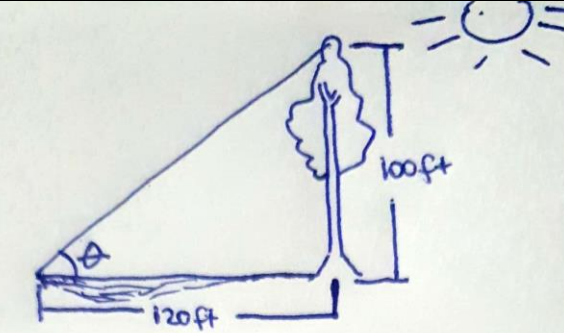


The angle of elevation of the object from the observer is α° .

PROBLEM SOLVING

1. A tree 100 feet tall casts a shadow 120 feet long. Find the angle of elevation of the sun.

i)


$$\tan \theta = \frac{\text{Opp}}{\text{adj}}$$
$$\tan \theta = \frac{100}{120} \quad \leftarrow \text{Take the inverse}$$
$$\theta = \tan^{-1}\left(\frac{100}{120}\right)$$

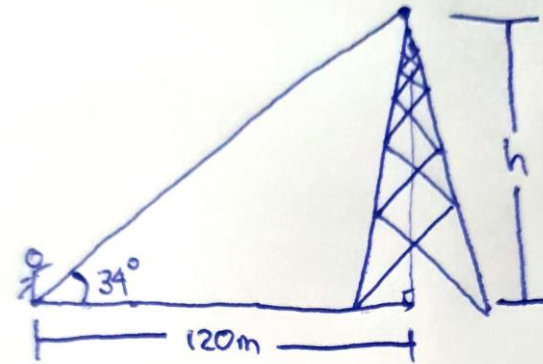
$\theta = 39.81^\circ$

//ans

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2. Johann is standing 120 meters from the base of a tower observes that the angle of elevation of the top of the pole is 34° . Find the height of the tower.

2.)



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 34^\circ = \frac{h}{120} \quad \leftarrow \text{cross multiply}$$

$$120 \tan 34^\circ = h \quad \leftarrow \text{rewrite}$$

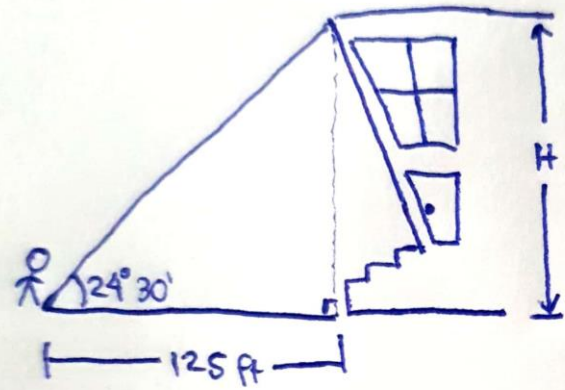
$$h = 120 \tan 34^\circ$$

$$\boxed{h = 80.94 \text{ m}}$$

Ans

3. On the way to a mall, Esther was told when she stand 125 feet from the base of the mall, the angle of elevation of its top is $24^{\circ}30'$. Find the height of the building.

3.)



note:
 $\theta = 24^{\circ}30'$
convert min to hours
read as:
24 degrees and 30 minutes

$$30 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 0.5 \text{ hrs}$$
$$\theta = (24 + 0.5)^{\circ}$$
$$\theta = 24.5^{\circ}$$
$$\tan 24.5^{\circ} = \frac{H}{125}$$
$$H = 125 \tan 24.5^{\circ}$$

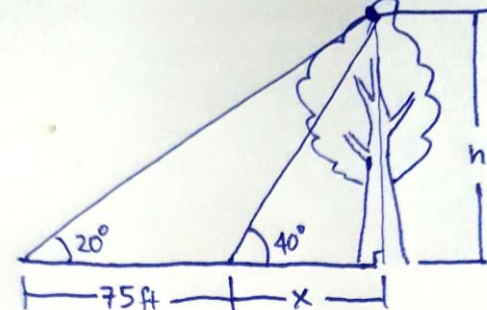
$H = 56.97 \text{ ft}$

ans

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4. Find the height of a tree if the angle of elevation of its top changes from 20° to 40° as the observer advances 75 feet towards the base.

4)



Larger Triangle Smaller Triangle



For two Unknown - needs two equations

for equation 1 for equation 2

$$\tan 20^\circ = \frac{h}{75+x}$$

$$\tan 40^\circ = \frac{h}{x}$$

$$(75+x) \tan 20^\circ = h$$

$$h = (75+x) \tan 20^\circ \rightarrow \text{eq 1}$$

$$x \tan 40^\circ = h$$

$$h = x \tan 40^\circ \rightarrow \text{eq 2}$$

equating ① & ②

equating ① & ②

$$h = h$$

$$(75+x) \tan 20^\circ = x \tan 40^\circ$$

$$75 \tan 20^\circ + x \tan 20^\circ = x \tan 40^\circ$$

$$75 \tan 20^\circ = x \tan 40^\circ - x \tan 20^\circ$$

$$75 \tan 20^\circ = x (\tan 40^\circ - \tan 20^\circ)$$

$$\frac{75 \tan 20^\circ}{0.475} = \frac{x (0.475)}{0.475}$$

$$x = \frac{75 \tan 20^\circ}{0.475}$$

$$x = 57.47 \text{ ft}$$

Solving for height subs. $x = 57.47$ to eqn. ②

$$h = 57.47 \tan 40^\circ$$

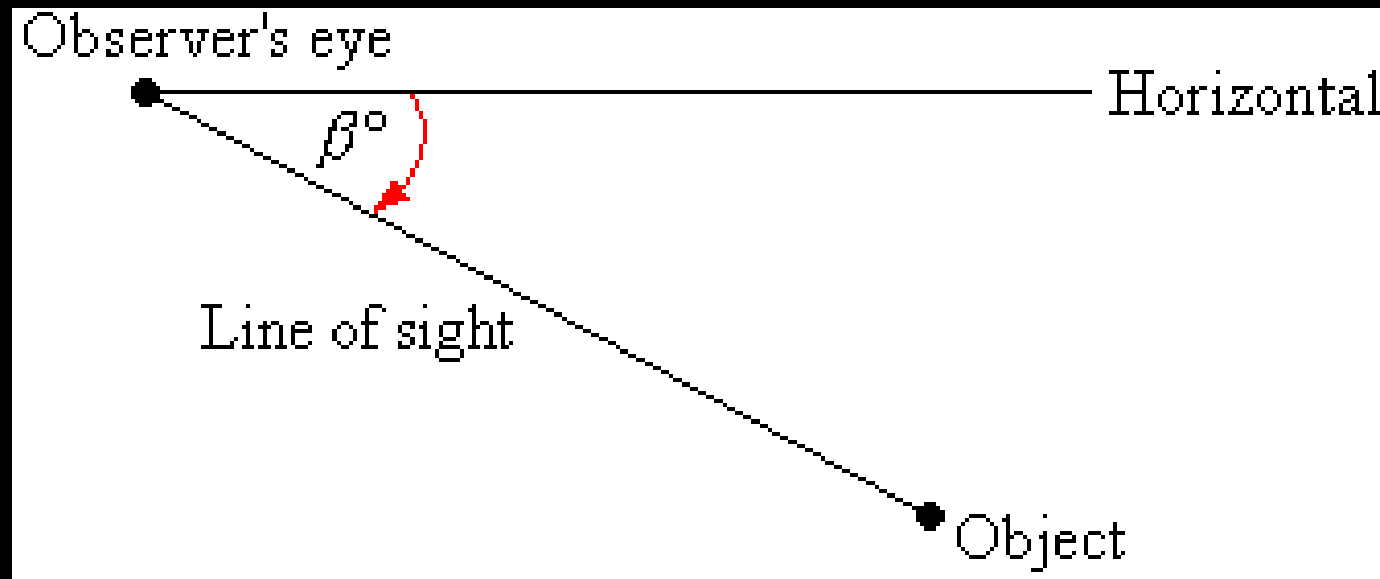
$$h = 48.22 \text{ ft} \quad \text{ans}$$

SEAT WORK #3

1. A vertical pole is 10 m from a building. When the angle of elevation of the sun is 40° , the pole cast a shadow on the building 1 m high, Find the height of the pole.
2. A flagstaff stands on the top of a house 15m high. From a point on the plane on which the house stands, the angles of elevation of the top and bottom of the flagstaff are found to be 60° and 45° respectively. Find the height of the flagstaff.

ANGLE OF DEPRESSION

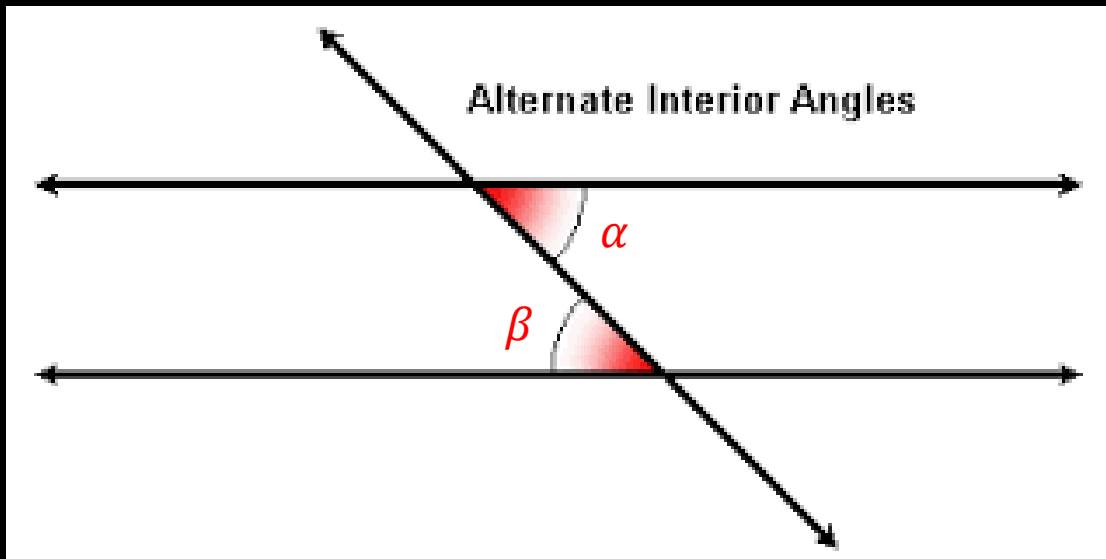
If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the **angle of depression**.



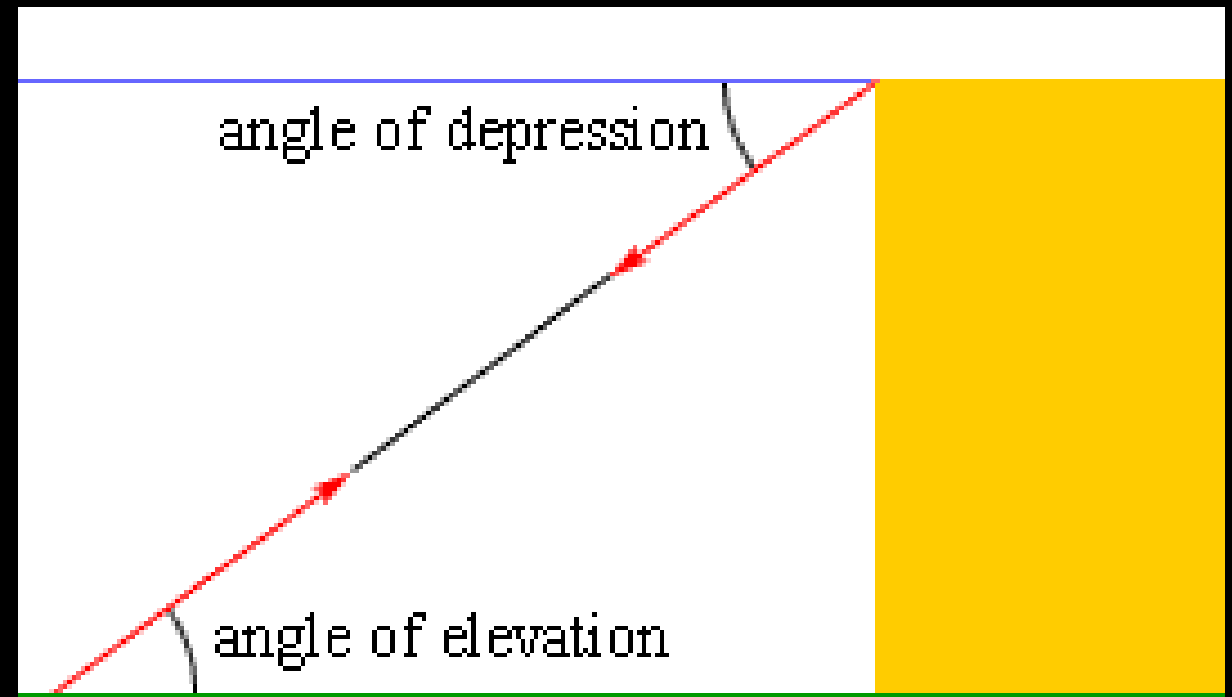
The angle of depression of the object from the observer is β° .

RELATIONSHIP OF ANGLE OF ELEVATION AND DEPRESSION

Considering the alternate interior angles



$$\alpha = \beta$$



Angle of elevation = Angle of depression

PROBLEM SOLVING

1. A guy wire 70 ft long is stretched from the ground to the top of a telephone pole 50 feet high. Find the angle between the wire and the pole.

1.)

$\cos \theta = \frac{\text{adj}}{\text{hyp}}$

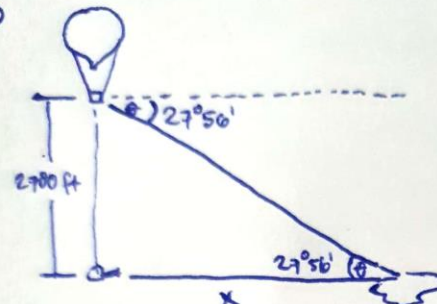
$\cos \theta = \frac{50}{70}$

$\theta = \cos^{-1}\left(\frac{50}{70}\right)$

$\theta = 44.415^\circ$ ans

2. An artillery observer in a captive balloon 2700 ft above his guns observes that the angle of depression of an enemy's fort is $27^{\circ}56'$. Find the distance from the guns to the fort. If they lie at the same elevations.

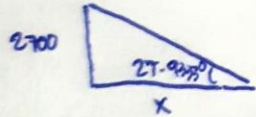
2.)



Recall alternate interior angles



$\theta = 27^{\circ}56'$
 $\theta = 27.9333^{\circ}$



$\tan 27.9333^{\circ} = \frac{2700}{x}$
 $x = \frac{2700}{\tan 27.9333^{\circ}}$
 $x = 5092.247 \text{ ft}$

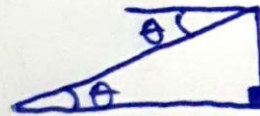
Mark

3. From a mountain 4000 ft above a fort, its angle of depression is $16^{\circ}45'$. Find the line-straight distance from the mountain to the fort.

3.)

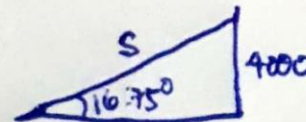


Recall alternate interior angles



$$\theta = 16^{\circ}45'$$

$$\theta = 16.75^{\circ}$$

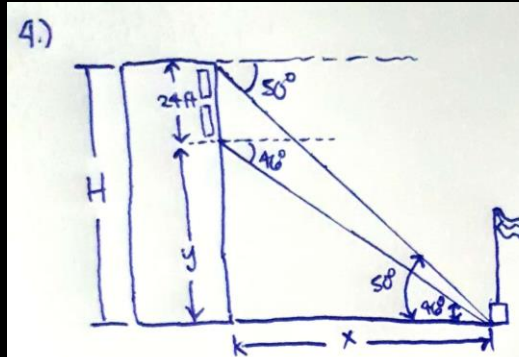


$$\sin 16.75^{\circ} = \frac{4000}{S}$$

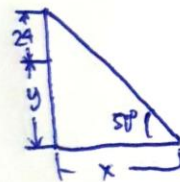
$$S = \frac{4000}{\sin 16.75^{\circ}}$$

$$S = 13,879.43 \text{ ft}$$

4. From the top of a building, the angle of depression of the base of the flagpole is 50° , while from the two floors below the angle of depression is 46° . if each of these floors extends 12 ft. Find the height of the building.



Larger Triangle



$$\tan 50^\circ = \frac{24 + y}{x}$$

$$x \tan 50^\circ = 24 + y$$

$$y = x \tan 50^\circ - 24 \rightarrow \text{eq. 1}$$

$$y = y$$

Smaller Triangle



$$\tan 46^\circ = \frac{y}{x}$$

$$x \tan 46^\circ = y$$

$$y = x \tan 46^\circ \rightarrow \text{eq. 2}$$

$$\begin{aligned} y &= y \\ x \tan 50^\circ - 24 &= x \tan 46^\circ \\ x \tan 50^\circ - x \tan 46^\circ &= 24 \\ x (\tan 50^\circ - \tan 46^\circ) &= 24 \\ x (0.156) &= \frac{24}{0.156} \\ x &= 153.85 \text{ ft} \end{aligned}$$

Substitute $x = 153.85$ to eq. 2

$$y = 153.85 \tan 46^\circ$$

$$y = 159.316 \text{ ft}$$

Solving for "H"

$$H = 24 + y$$

$$H = 24 + 159.316$$

$$\boxed{H = 183.32 \text{ ft}} \text{ ans}$$

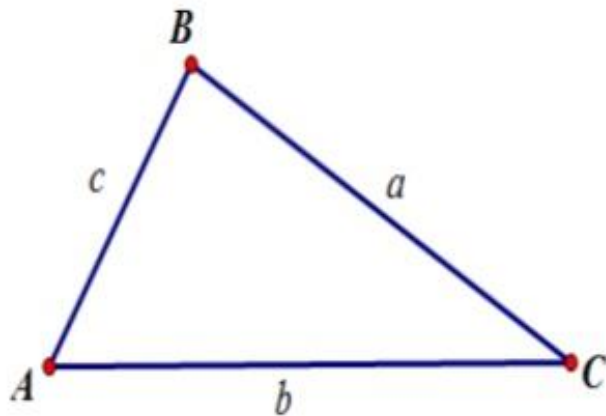
SEAT WORK #4

1. Point A and B are on the same horizontal line with the foot of a hill and the angles of depression of these points from the top of the hill are 32° and 22° , respectively. If the distance between A and B is 80 meters, what is the height of the hill?
2. From a cliff 150 feet above a lake, we see a boat sailing directly towards us. The angle of depression of the boat is seen to be $5^\circ 7'$ and then later is $11^\circ 18'$. Find the distance sailed between observations.

APPLICATION ON OBLIQUE TRIANGLES

- A. Law of Sine
- B. Law of Cosine

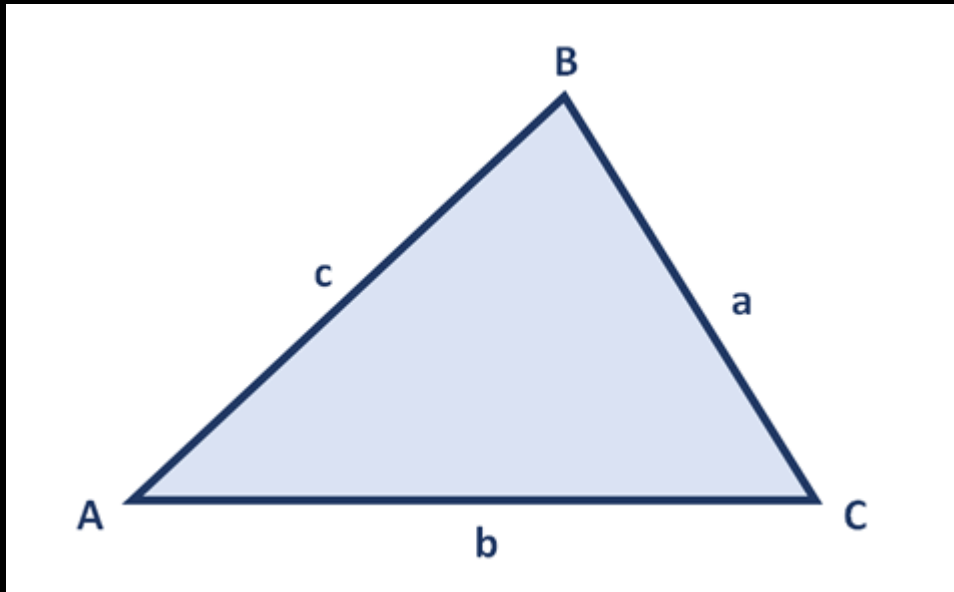
Oblique Triangle



- | | |
|---------------------|------------|
| 1. Given: A, b, C | Law of Sin |
| 2. c, B, C | Law of Sin |
| 3. c, a, C | Law of Sin |
| 4. b, A, c | Law of Cos |
| 5. c, B, a | Law of cos |
| 6. a, b, c | Law of cos |

THE SINE LAW

In any given triangle, the ratio of the length of a side and the sine of the angle opposite that side is a constant.



Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

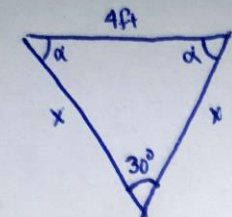
The Law of Sines is also known as the sine rule, sine law, or sine formula. It is valid for all types of triangles: right, acute or obtuse triangles.

The Law of Sines can be used to compute the remaining sides of a triangle when two angles and a side are known (AAS or ASA) or when we are given two sides and a non-enclosed angle (SSA).

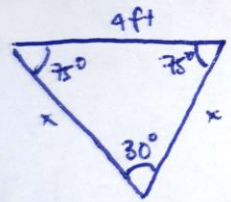
PROBLEM SOLVING

1. A curtain on the window was tied at the bottom part forming an inverted isosceles triangle. The angle at the vertex located in the part is 30° and its opposite side 4ft. Find the length of the remaining sides.

1.)



Solving for α

$$180^\circ = 30^\circ + \alpha + \alpha$$
$$180^\circ = 30^\circ + 2\alpha$$
$$180^\circ - 30^\circ = 2\alpha$$
$$\frac{150^\circ}{2} = \frac{2\alpha}{2}$$
$$\alpha = 75^\circ$$


Sine Law

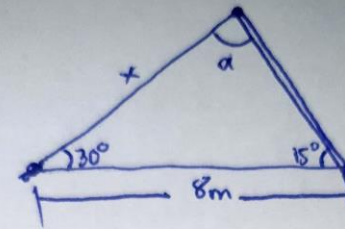
$$\frac{4}{\sin 30^\circ} = \frac{x}{\sin 75^\circ}$$
$$\frac{4 \sin 75^\circ}{\sin 30^\circ} = x$$

$x = 7.73$

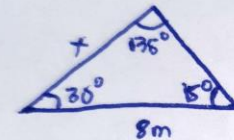
ft

2. A pole stands on a plane which makes an angle of 15° with the horizontal. A wire from the top of the pole is anchored on a point 8 m from the foot of the pole. If the angle between the wire and the horizontal plane is 30° , find the length of the wire?

2.)



$$\begin{aligned} 180^\circ &= 30^\circ + 15^\circ + \alpha \\ 180^\circ &= 45^\circ + \alpha \\ \alpha &= 135^\circ \end{aligned}$$



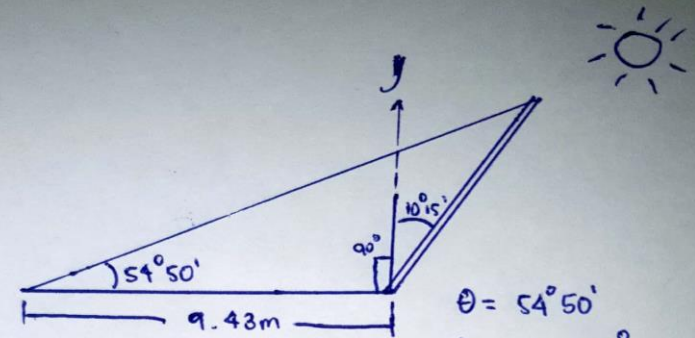
$$\frac{8}{\sin 135^\circ} = \frac{x}{\sin 15^\circ}$$

$$\frac{8 \sin 15^\circ}{\sin 135^\circ} = x$$

$$x = 2.93 \text{ m}$$

3. A pole which leans to the sun by $10^{\circ}15'$ from the vertical casts a shadow of 9.43m on the level ground when the angle of elevation of the sun is $54^{\circ}50'$. Find the length of the pole.

3.)



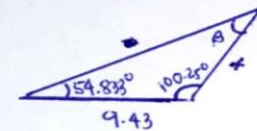
$$\theta = 54^{\circ}50'$$

$$\theta = 54.833^{\circ}$$

$$\alpha = 90^{\circ} + 10^{\circ}15'$$

$$\alpha = 90^{\circ} + 10.25^{\circ}$$

$$\alpha = 100.25^{\circ}$$



$$\beta = 180^{\circ} - (54.833^{\circ} + 100.25^{\circ})$$

$$\beta = 24.917^{\circ}$$

Sine Law

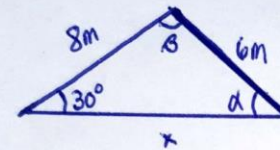
$$\frac{9.43}{\sin \beta} = \frac{x}{\sin 54.833^{\circ}}$$

$$\frac{9.43 \sin 54.833^{\circ}}{\sin 24.917^{\circ}} = x$$

$$x = 18.3 \text{ m}$$

4. Two poles 6 meters high support a clothesline which is 8 meters long. The other one fell on the ground and created a 30° angle, also causing the other pole to bend. Find the distance between the two bases of the poles on the ground.

4.)



sine law
solving α

$$\frac{6}{\sin 30^\circ} = \frac{8}{\sin \alpha}$$

$$\sin \alpha = \frac{8 \sin 30^\circ}{6}$$

$$\sin \alpha = 0.6667$$

$$\alpha = \sin^{-1}(0.6667)$$

$$\alpha = 41.8^\circ$$

$$\beta = 180^\circ - (30^\circ + 41.8^\circ)$$

$$\beta = 108.2^\circ$$

sine law
solving x

$$\frac{x}{\sin \beta} = \frac{6}{\sin 30^\circ}$$

$$x = \frac{6 \sin 108.2^\circ}{\sin 30^\circ}$$

$$x = 11.4 \text{ m}$$

THE COSINE LAW

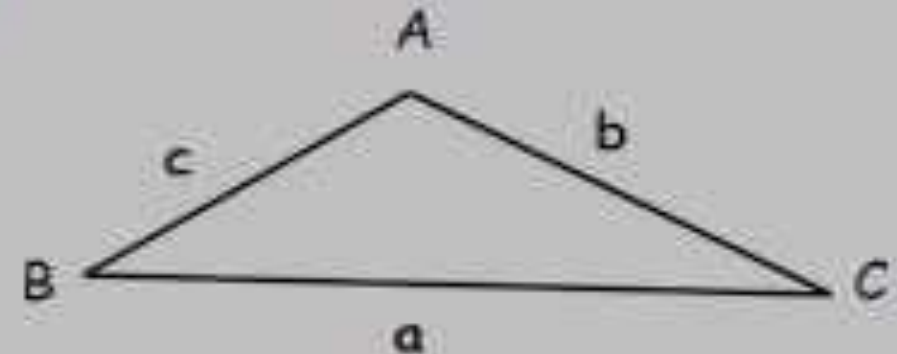
The Law of Cosines relates the lengths of the sides of a triangle with the cosine of one of its angles.

The Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



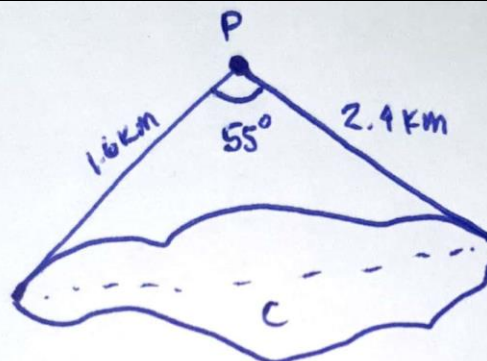
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \quad \text{or} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

If we are given two sides and an included angle (SAS) or three sides (SSS) then we can use the Law of Cosines to solve the triangle i.e. to find all the unknown sides and angles.

PROBLEM SOLVING

1. A point P is 1.6 km from one end of the lake and 2.4 km from other end. If at P the lake subtends an angle of 55° , what is the length of the lake?

1.)



cosine law

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$c^2 = (1.6)^2 + (2.4)^2 - 2(1.6)(2.4) \cos 55^\circ$$
$$\sqrt{c^2} = \sqrt{3.9204}$$

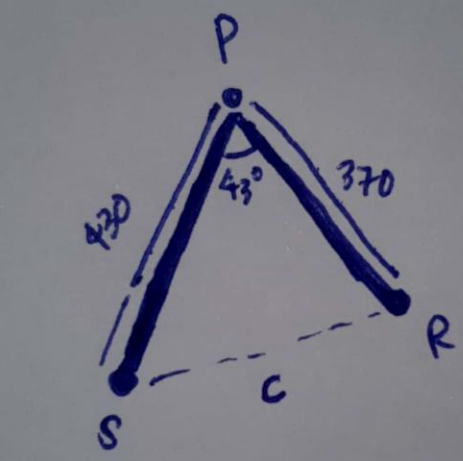
$c = 1.98 \text{ m}$

Ans

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2. Two straight roads intersect at a point P and makes an angle of 43° there. At a point R on the road is a building that is 370 meters from the P and at point S on the other road is a building that is 430 meters from P. Determine the direct distance from R to S.

2.)



cosine law

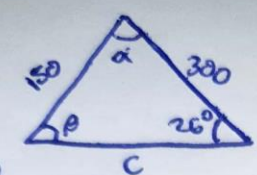
$$c^2 = (430)^2 + (370)^2 - 2(430)(370) \cos 43^\circ$$
$$\sqrt{c^2} = \sqrt{89083.25}$$
$$c = 298.47 \text{ m}$$

Ans

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3. The two legs of the triangle are 300 cm and 150 cm respectively. The angle opposite the 150 cm side is 26° . Find the length of the third side?

3.)



sine law

$$\frac{300}{\sin \beta} = \frac{150}{\sin 26^\circ}$$

$$\frac{300 \sin 26^\circ}{150} = \sin \beta$$

$$0.8767 = \sin \beta$$

$$\beta = \sin^{-1}(0.8767)$$

$$\beta = 61.25^\circ$$

$$\alpha = 180^\circ - (61.25^\circ + 26^\circ)$$

$$\alpha = 92.75^\circ$$

cosine law

$$c^2 = (300)^2 + (150)^2 - 2(300)(150) \cos (92.75^\circ)$$

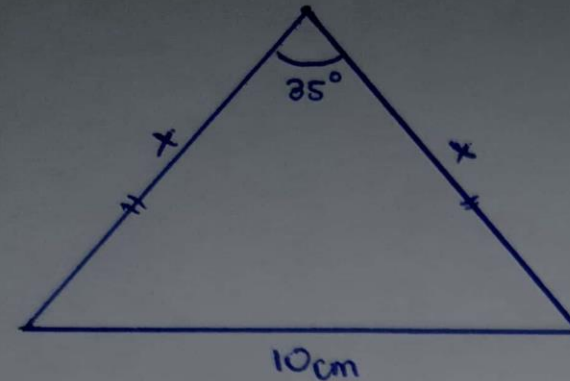
$$\sqrt{c^2} = \sqrt{116818.032}$$

$$c = 341.79 \text{ cm}$$

Ans

4. The vertex angle of an isosceles triangle is 35° . The length of the base is 10 cm. How many centimeter are there in the perimeter.

4.)



cosine law

$$c^2 = a^2 + b^2 - 2ab \cos 35^\circ$$

$$c^2 = x^2 + x^2 - 2(x)(x) \cos 35^\circ$$

$$c^2 = 2x^2 - 2x^2 \cos 35^\circ$$

$$c^2 = x^2 (2 - 2 \cos 35^\circ)$$

$$\frac{10^2}{0.362} = \frac{x^2 (0.362)}{0.362}$$

$$276.48 = x^2$$

SEAT WORK #5

1. The side of the triangle ABC are $AB = 5$, $BC = 12$, And $AC = 10$. Find the length of the line segment drawn from vertex A and bisecting BC.
2. A tower 150 m high is situated at the top of a hill. At point 650 m down the hill and the line of sight to the top of the towers is $12^{\circ}30'$. Find the inclination of the hill to a horizontal plane.
3. A 50 ft electrical pole (vertical) casts a 62.3 ft length shadow when the angle of elevation of the sun is 41.6° . Determine the inclination of the ground.
4. A tree leans at an angle of 8° from the vertical away from the sun. From a point 8 m from the bottom of the tree, the angle of elevation to the top of the tree is 66° . How tall is the tree?

ASWER KEY SW

SW #3

1. 9.39 ft
2. 10.98 m

SW #4

1. 91.46 m
2. 924.53 ft

SW#5

1. 4.1
2. $7^{\circ}50'$
3. 4.72°
4. 13.79